

On the modelling of time-frequency dependent power spectral density function of the turbulent winds from tropical cyclones and thunderstorms

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SUMMARY:

The turbulent winds for tropical cyclones and thunderstorm events are frequently modelled as a uniformly modulated evolutionary process with time-invariant or time-varying mean wind speed. We argue that the use of uniform modulation is insufficient, and the amplitude and frequency modulated evolutionary stochastic process should be considered for such turbulent winds. We provide some theoretical basis for such a consideration. The theoretical derivation is based on the concept of time transformation and the instantaneous power spectrum representation resulting and amplitude and frequency modulation. We also give the time-frequency decomposition results for the recorded wind from tropical cyclones and thunderstorm events. These results are used to further support the argument that the fluctuating wind component for such wind events should be modelled as amplitude and frequency modulated evolutionary stochastic process. Furthermore, we discuss whether the wind time history for the fluctuating wind should be modelled as Gaussian or non-Gaussian and its implication for its simulation.

Keywords: Time-frequency decomposition, turbulent wind, power spectral density function

1. INTRODUCTION

Wind speeds are often represented as the superposition of the mean and fluctuating components. For synoptic winds, the mean wind component is treated as time-invariant and the fluctuating (or turbulent) wind component is modelled as a stationary stochastic process characterized by its power spectral density (PSD) function (Simiu and Scanlan 1996). The wind speed from the tropical cyclones and thunderstorm events is often considered to be nonstationary. The nonstationary could be due to time-varying mean wind speed, the time-varying intensity or amplitude of the turbulent wind or the time-frequency dependent PSD (TFPSD) function. For practical applications, the nonstationary turbulent winds are models as an evolutionary process.

The physical aspects of the decomposition of the recorded winds into the low frequency components (i.e., mean wind) and high frequency components (i.e., turbulent wind) are given in Di Paola (1998), Kareem (2008), and Tubino and Solari (2022). The moving average, windowed Fourier transform, wavelet functions, and empirical modal decomposition technique could be employed for the decomposition (Choi and Hidayat 2002; Chen and Letchford 2007; Solari et al. 2015; Su et al. 2015; Huang et al. 2015; Tubino and Solari 2020). Several authors considered that the nonstationary turbulent wind could be modelled as a uniformly modulated (e.g., time-varying standard deviation) evolutionary process. This implicitly assumes that the frequency content of the

fluctuating winds does not change with the time-varying mean wind speed. It was argued in Hong (2016) that such an assumption is inconsistent with that the frequency content depends on the mean wind speed as in the von Karman spectrum, Davenport spectrum, and Kaimal spectrum. Moreover, by using the hurricane (i.e. tropical cyclone (TC)) wind record (Balderrama et al. 2011), Xiao and Hong (2022) showed that there is clear time-varying frequency content of the turbulent TC winds. Furthermore, by decomposing using the recorded thunderstorm winds in the time-frequency (or time-scale) domain (i.e., applying the S-transform and continuous wavelet transform), it was shown (Hong et al. 2021, 2022; Liu and Hong 2022) that there is time-varying frequency content of the turbulent thunderstorm winds.

The main objectives of this study are to put, in a succinct manner, the arguments for supporting the need to model the turbulent wind component from tropical cyclones and thunderstorms with amplitude and frequency modulation.

2. MODELING NONSTATIONARY WIND

2.1. A theoretical model based on time transform

It is well known that the wind spectrum for stationary turbulent wind $u(t)$ can be expressed in terms of reduced frequency $\zeta = fz/U(z)$, where f Hz represents the frequency, z is the height above the ground surface and $U(z)$ is the mean wind speed (Simiu and Scanlan 1996). For example, by letting $\tau = (U(z)/z)t$ representing a dimensionless time and $\tilde{u}(\tau) = u(t)/\sigma(z)$, the Kaimal spectrum for $\tilde{u}(\tau)$, $S_{\tilde{u}}(\zeta)$, can be written as (Hong 2016),

$$S_{\tilde{u}}(\zeta) = (200/6) / (1 + 50\zeta)^{5/3}. \quad (1)$$

The PSD function of $u(t)$, $S_u(n)$, and $S_{\tilde{u}}(\zeta)$ is related by

$$S_u(f) = \sigma^2(z) \times [1/(U(z)/z)] S_{\tilde{u}}(\zeta). \quad (2)$$

Consider the turbulent wind with a time-varying mean wind speed and standard deviation, denoted as $U(z,t)$, and $\sigma(z,t)$. Let $\tau(t) = [\int_0^t U(z,\hat{t})d\hat{t}] / z$ denote the time transform by considering time-varying $U(z,t)$. It can be shown that the PSD function of $u(t) = \sigma(z,t) \times \tilde{u}(\tau(t))$ is given by (Yeh and Wen 1990; Hong 2016),

$$S_u(f,t) = (\sigma(z,t))^2 \times [1/\tau'(t)] S_{\tilde{u}}(f/\tau'(t)) \quad (3)$$

where $\tau'(t)$ represents the first derivative with respect to t . $S_u(f,t)$ is time-varying and depends on $\tau'(t)$. If $U(z,t) = U(z)$ and $\sigma(z,t) = \sigma(z)$, this model leads to $\tau'(t) = U(z)/z$ and $S_u(f,t)$ to be identical to $S_u(f)$ shown in Eq. (2), indicating that the model reduces to the stationary case with well-known PSD function. In general, the above shows that the application of the defined time transform (i.e., $\tau \rightarrow t$) for the stationary $\tilde{u}(\tau)$ (in τ -domain) results in a nonstationary process in the t -domain with the TFPSD function given by Eq. (3), which differs from that for a uniformly modulated evolutionary process because there is frequency modulation.

2.2. Time-frequency decomposition of tropical cyclone wind

The analysis of the recorded tropical cyclone winds has been presented in the literature (see references in Xiao and Hong (2022)). By removing the time-varying mean wind speed (i.e., low frequency wind components) of the recorded TC winds (Balderrama et al. 2011) and carrying out the time-frequency decomposition using the S-transform (ST) (Stockwell et al. 1996) with s denoting the localization time in ST, Xiao and Hong (2022) showed that the typical TFPSD

function for the turbulent TC wind is as shown in Figure 1f. Figure 1 indicates that there is clear amplitude and frequency modulation. Most importantly, a fitting analysis indicates that the TFPSD function in Figure 1f could be approximated by using,

$$fS_s(f, s) / \sigma^2(s) = A\zeta / (C + B\zeta^\gamma)^{(5/3)\gamma}, \quad (4)$$

where A , C , B , and γ are parameters to be determined (Olsen et al. 1984). This indicates that the nonstationary turbulent TC wind could be viewed as the transformed stationary process with a PSD function shown in Eq. (4).

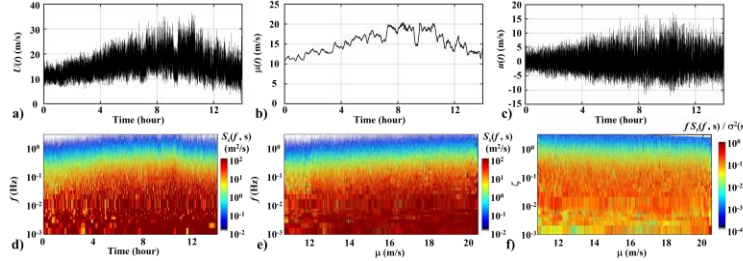


Figure 1. Characteristics of recorded TC wind at site Kure Beach, NC, Tower 0 for the hurricane Dennis in 1999 (see Balderrama et al. 2011): *a*) wind record, *b*) mean $\mu(t)$ obtained by using a moving window of 600 s, *c*) wind speed with mean removed, $u(t)$, and *d*) TFPSD presented in chronological order, *e*) TFPSD presented as a function of the value of $\mu(t)$, μ , *f*) standardized TFPSD presented as a function of μ . ($S_s(f, s)$ denote the TFPSD obtained using ST; $\sigma(s)$ is the standard deviation at the localization time s) (after Xiao and Hong (2022) replotted).

2.3. Time-frequency decomposition of recorded thunderstorm wind

The analysis of the thunderstorm winds has been carried out in the literature as indicated in the introduction section. By processing the thunderstorm wind records that were given by Burlando et al. 2018, removing the time-varying mean, and utilizing the continuous wavelet transform with the harmonic wavelet (CWT-HW) to decompose the turbulent thunderstorm winds, Liu and Hong (2022) presented the spectral analysis results.

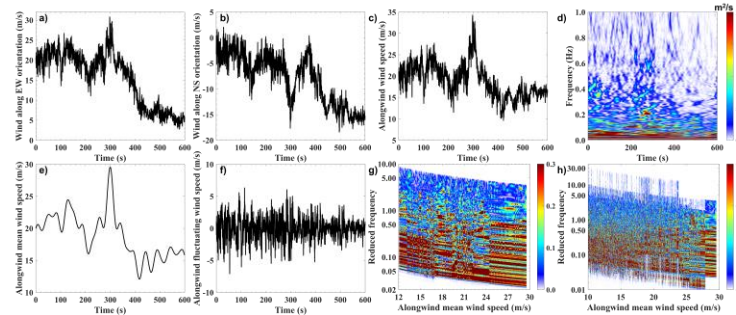


Figure 2. *a*) & *b*) Recorded wind in the EW and NS orientations (Livorno 01, 2011.12.16, see Burlando et al. (2018)); *c*) & *d*) wind speed in the alongwind direction and its corresponding TFPSD function; *e*) & *f*) mean and turbulent wind speed in the alongwind orientation; *g*) standardized TFPSD function of the turbulent wind; *h*) standardized TFPSD of turbulent wind obtained using recorded winds from 26 thunderstorm events.

A typical example of the obtained results is presented in Figure 2, where *2a*) and *2b*) show the recorded winds, *2c*) depicts the obtained wind speed in the time-dependent alongwind orientation, and *2d*) shows the TFPSD function of the alongwind depicted *2c*). *2e*) shows the mean wind speed (i.e., wind components with a frequency $< 1/30$ Hz) extracted from *2d*). The wind speed for winds with a frequency $> 1/30$ Hz (i.e., turbulent winds) is shown in *2f*). The evaluated TFPSD function of the turbulent winds by using CWT-HW is shown in *2g*) as a function of the mean wind speed. The analysis that is carried out for *2a*) to *2g*) is repeated for thunderstorm winds from 26 thunderstorm events with the obtained TFPSD function shown in *2h*). A fitting exercise indicates that for a given mean wind speed value of $\mu(t)$, the TFPSD function can be adequately

approximated by using the parametric form shown in the right-hand side of Eq. (4). However, because of the sampling frequency and the removal of the mean using a cut-off frequency of 1/30 (Hz), an upper and lower bound for ζ , denoted as ζ_L and ζ_U should be imposed for the fitted empirical model.

3. CONCLUSIONS

In this study, we showed, theoretically, that the application of the defined time transform to a stationary wind (in τ -domain) results in a nonstationary process in the t -domain with the TFPSD function that includes both the amplitude and frequency modulation. This differs from the PSD function for a uniformly modulated evolutionary process (i.e., no frequency modulation). Results of the time-frequency decomposition analysis by using ST and CWT-HW show that the energy distribution of the turbulent TC and thunderstorm winds include amplitude and frequency modulations. Therefore, rather than model such winds as a uniformly modulated evolutionary process, use of amplitude and frequency modulated nonstationary process should be considered.

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